ECON 53015-
Advanced Economic Theory- Microeconomics

# Theory of Production

## Long-Run Analysis of Production

Prof. W. M. Semasinghe

# Introduction

- Long-run production analysis concerned about the producers' behavior in the long-run.
- In the long-run, expansion of output can be achieved by varying all factors.
- In general, in the long-run, producers can change its scale of production
- Since, all factors can be changed together there is no fixed factors in the long-run. All are variable factors.
- In the long-run, output can be increased by changing all factors by the same proportion or by different proportions. Traditional theory of production concentration on the first case i.e. change of output as a result of change of all inputs change by same proportion.

Long-run production process is subjected to the *Laws of Returns to Scale*.

Laws of returns to scale describes the effects of change of all inputs together OR effect of change of scale of production, on the level of output.

There are 3 types of Returns to Scale:

| | |
|---|---|
| Constant Returns to Scale | With increase all inputs together (scale of production increases) by some proportion, output increases by same proportion. |
| Increasing Returns to Scale | With increase all inputs together (scale of production increases) by some proportion, output increases by greater proportion. |
| Decreasing Returns to Scale | With increase all inputs together (scale of production increases) by some proportion, output increases by smaller proportion. |

1. Consider production function $Q = 5L^{0.5}K^{0.3}$. What type of returns to scale does it exhibit?

2. Consider production function $Q = 10L^{0.5}K^{0.5}$. What type of returns to scale does it exhibit?

3. Suppose you find that in a certain industry which uses capital and labor is subjected to the production function $Q = L^{0.75}K^{0.25}$. Will it be right to say that in the industry, output per worker will be a function of capital per worker?

# Economies of scale of Production

What are the reasons behind the returns to scale?
As economists explain, broadly, the reasons for returns to scale are

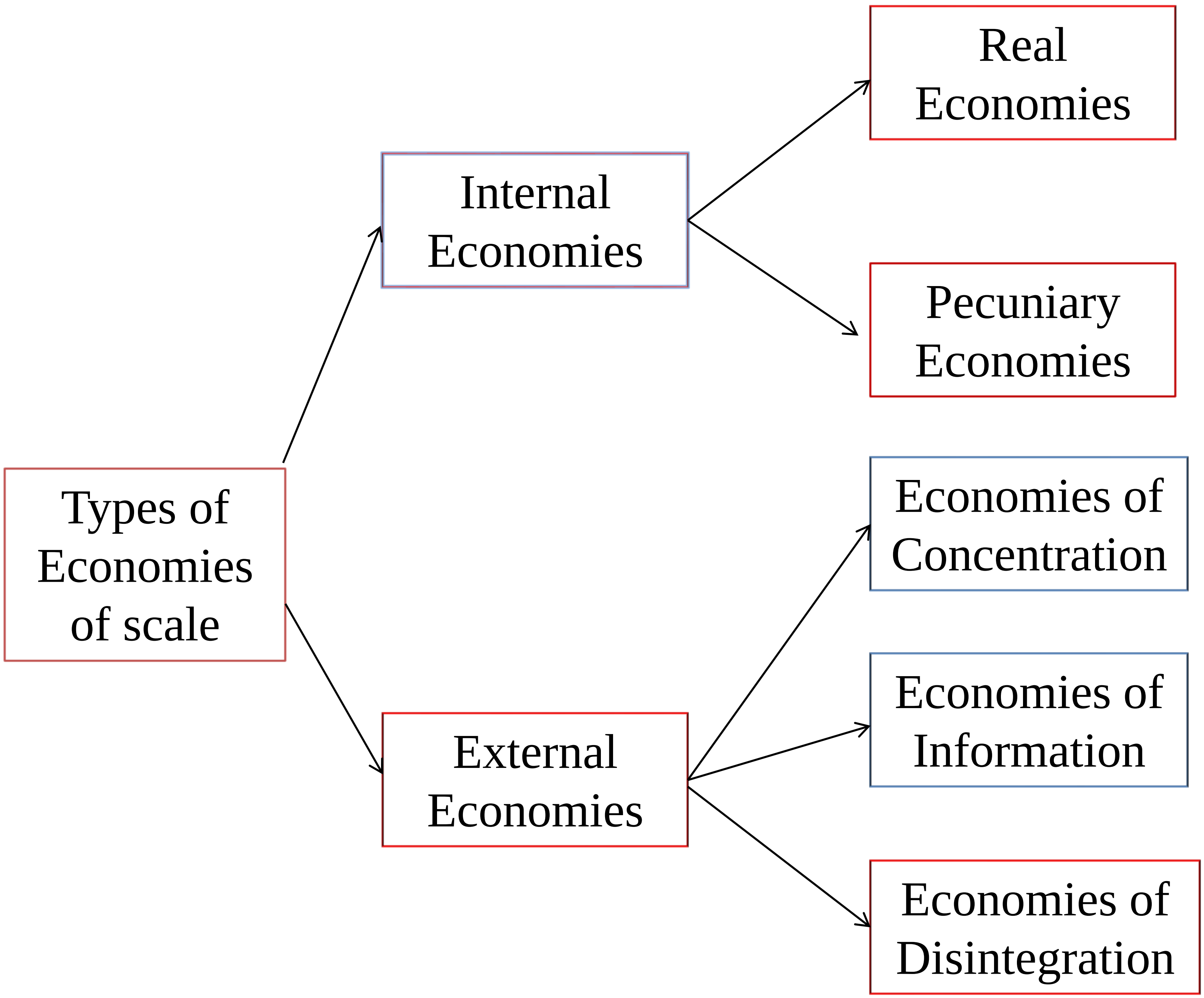
i. division of labor and specialization and

ii. technological factors related to the production.

When the scale of production increases, the firm gets the advantages of these factors.

The advantages the production process gain due to these factors are called Economies of scale.

Increasing returns to scale is the result of these economies of scale. When scale of production increases up to a certain point, the firm/producer gets these economies.


Types of Economies of scale
Internal Economies
External Economies
Real Economies
Pecuniary Economies
Economies of Concentration
Economies of Information
Economies of Disintegration

# Internal Economies

- Internal economies are those which are open to a single firm or a producer independently of the actions of other firms. They result from an increase in the scale of output of a firm and cannot be achieved unless increase of scale.

## Real Economies of Scale

- Real economies are those associated with a reduction in the physical quantity of inputs, raw materials, various types of labor and various types of capital in average.

There are several types of Real Economies:

- Labor Economies
- Technical Economies
- Inventory Economies
- Marketing Economies
- Managerial Economies
- Transport & storage Economies

## Labor Economies

Increase in scale of production results into the following Economies of Labor:

- Specialization
- Time Saving
- New Inventions
- Automation of Production Process

## Technical Economies

These Economies influence the size of the firm. These result from greater efficiency of the capital goods employed by the firm. These are following types:

- Economies of increased dimensions
- Economies of linked processes
- Economies of use of by-product.

## Inventory Economies

A large-sized firm enjoys several types of Inventory Economies such as:

- Large stock of raw materials
- Large stock of spare parts & small tools

As such there is no fear of stoppage of production.

## Marketing Economies

A large-sized firm enjoys several types of Marketing Economies such as:

- Economies on account of advertisement
- Appointment of sole distributers & Authorized dealers
- Economies of account of Research and Development

All these enables the firm to produce quality Products.

## Managerial Economies

With the increase in production, management cost will reduce as a result of:

- Appointment of Efficient and Talented Managers,
- Decentralization of Task.

## Transport and Storage Economies

- Own transportation system
- Own storage and go-down facilities

With these, the firm/producer is able to sell its product at the opportunity time and at favorable price.

# Pecuniary Economies of Scale

Pecuniary Economies are economies that realized from paying lower prices for the factors used in production and distribution due to bulk buying by the firm as its size increased.

Example

- Procurement of raw material at lower prices,
- Concessional loans from Bank,
- Large discounts & commissions on advertisement & publicity of their products etc.

# External Economies of Scale

External Economies of Scale refers to all those benefits and facilities which are available to all the firms in a given industry. The following three are External Economies:

- Economies of Concentration
- Economies of Information
- Economies of Disintegration

## Economies of Concentration

When several firms of an industry establish themselves at one place, then they enjoy many benefits together.

- Availability of developed means of communication and transport
- Trained labor
- Development of new inventions pertaining to that industry etc.

## Economies of Information

When the number of firm in an Industry increases, it becomes possible for them to have concerted efforts and collective activities such as publication of scientific & trade journals providing sundry information to the firm of a given industry.

## Economies of Disintegration

When an industry develops, the firms engaged in it mutually agree to divide the production process among them. Every firms specialize in the production of a particular item concerning that industry.

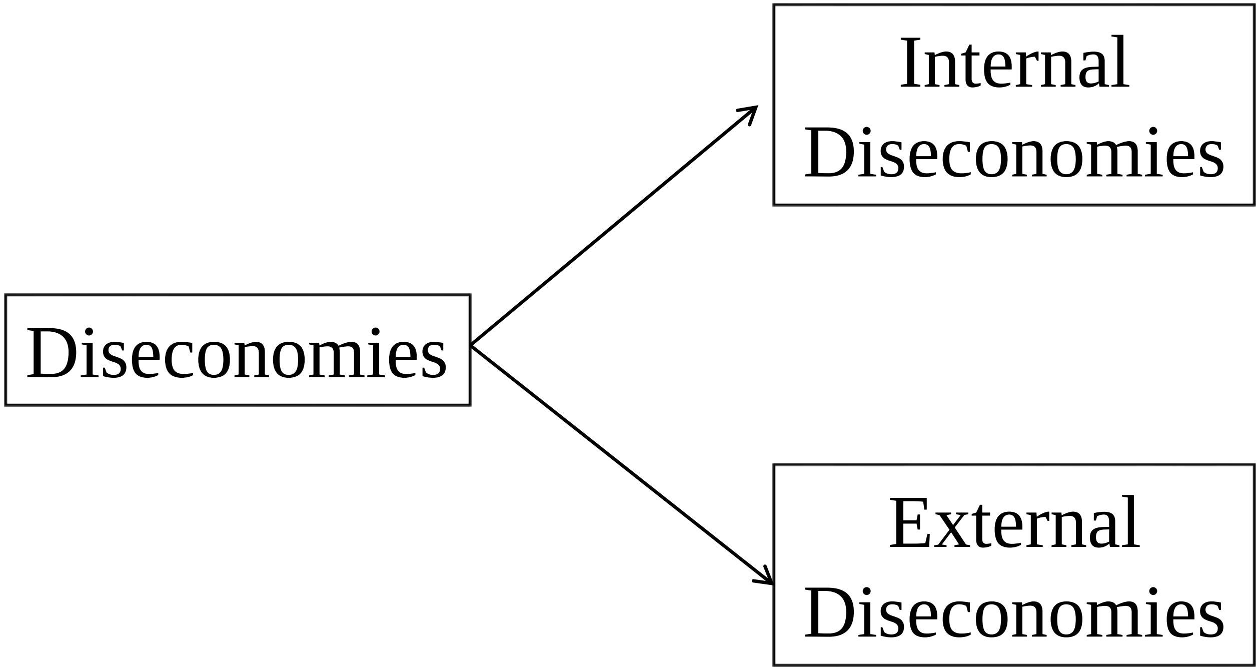
# Diseconomies of Scale

- The economies of scale which yield increasing returns to scale would not continue for a long-term when inputs increase further and further. The size of production will unmanageable and so inefficiency creeps. Moreover, it may be difficult to get certain critical inputs in the same proportions as others.

- The disadvantages that the production process get due to the expansion of its scale is called diseconomies of scale.

- Decreasing returns to scale is the result of these diseconomies.

There are two types of  Diseconomies:



## Internal Diseconomies of Scale

- Internal diseconomies arise when a given firm increases its scale of production beyond a certain point.

- Internal diseconomies arise because of 2  reasons:

    - Unwieldy Management
    - Technical Difficulties.

- Internal diseconomies are limited for a given firm but not affect the entire industry.

# External Diseconomies of Scale

These diseconomies are suffered by all the firms in an industry irrespective their scale of output.

e.g. When many firms are located at a particular place, then it becomes difficult for means of transport creating additional burden of traffic, and hence transport cost goes up.

# Iso-quant Analysis of Production

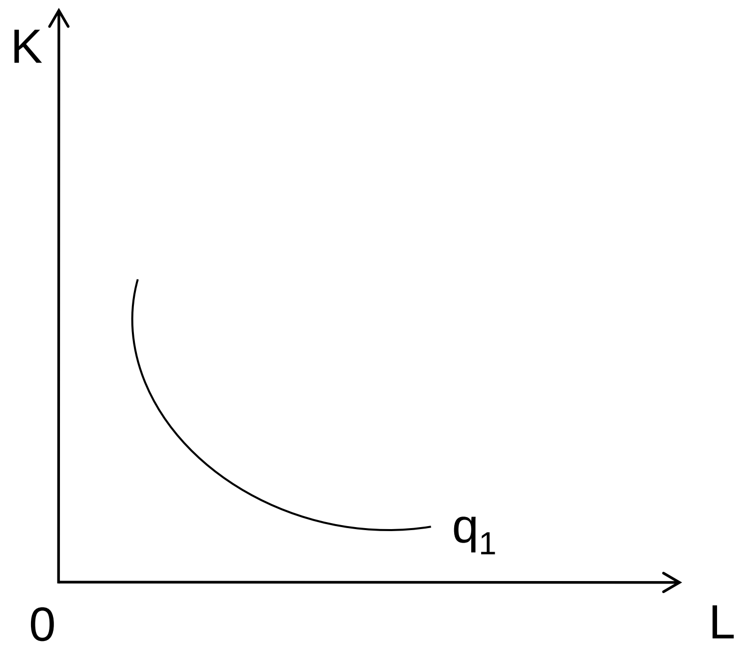
Behavior of the firms/producers in the Long-run production process can be analyzed by using Iso-quant analysis.

Basic assumptions of the analysis:

- Level of output remains constant
- Inputs are divisible and can be substituted for each other.

Iso-quant can be take the form of a schedule, a graph or an equation. Two inputs iso-quant can be depicted with the familiar production function framework as $q = f(L,K) = k$

# Graphical representation of Iso-quant




An Iso-quant

*"Iso-quant is the locus of all possible combinations of the inputs which yield a specific level of output".*

The shape of isoquants may be convex towards the origin as given in above graphs or straight lines or right-angled-shaped depending on the substitutability among factors of production.

# Equilibrium of the Firm: Choice of Optimal Combination of Factors of Production

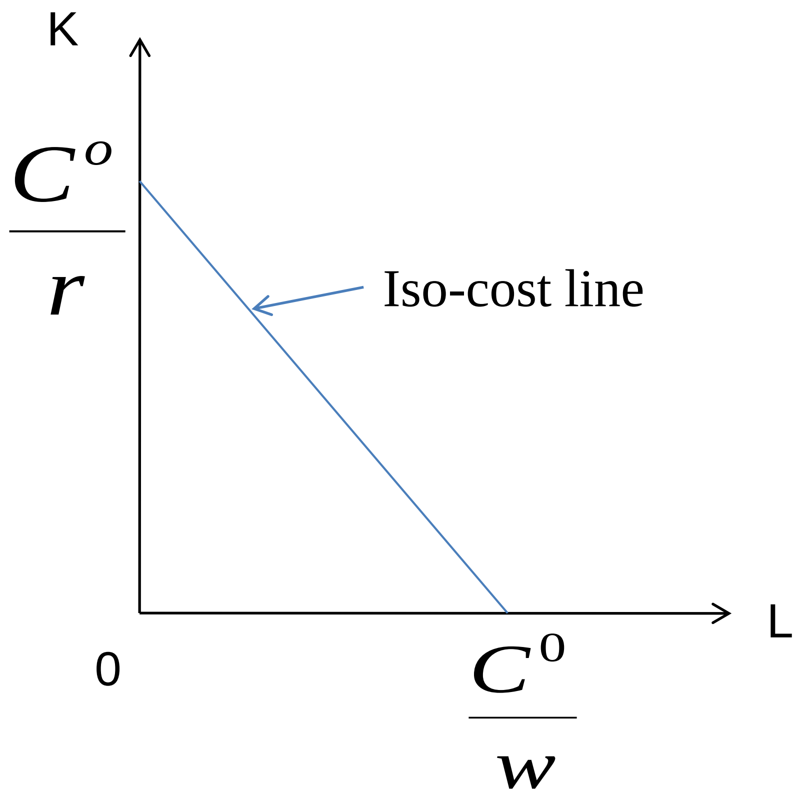
What is the actual combination of the inputs to be used to produce given level of output?

To take such decision it is required information about total outlay of the producer and the factor prices.

If the producer has $C^0$ units of money to spend on the inputs. Suppose $w$ and $r$ are the prices of a unit of L and K respectively and $C^0$, $w$ and $r$ are fixed:

Producer's budget equation or resource line i.e. *Iso-cost line,* can be defined as

$$C^0 = wL + rK$$




Slope of the Iso-cost line = $\frac{w}{r}$

Given these information producer would be able to determine the optimum input combination in two ways:

i. Constrained output maximization i.e. maximizing output for a given cost.

ii. Constrained cost minimization, i.e. minimizing cost subject to a given output.

In all these cases we make the following assumption:

i. The goal of the firm is profit maximization, i.e. maximization of the difference between TR and TC.

$$\Pi = TR - TC$$

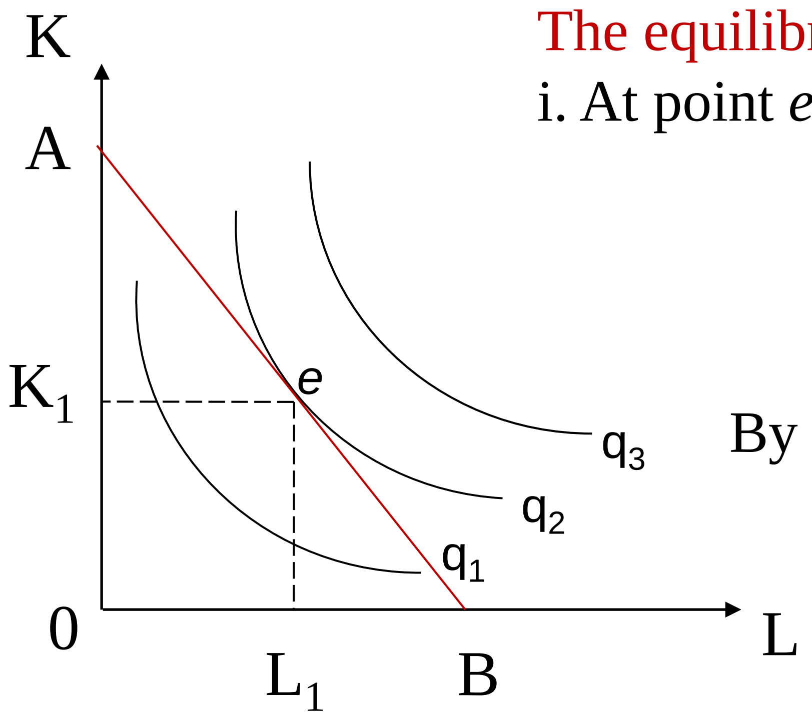
ii. The price of output is given

iii. The prices of factors are given

## Constrained output maximization – Graphical approach

PF or the equation of the iso-quant is Q = f(L,K) and prices of L and K are *w* and *r*, respectively.

Graphically, optimum factor combination can be defined by the tangency of iso-cost line and highest possible iso-quant.




The equilibrium conditions are:

i. At point *e*, slope of iso-quant = slope of iso-cost line

$$\frac{MP_L}{MP_K} = \frac{w}{r} = MRTS_{L,K}$$

By rearranging this, $\frac{MP_L}{w} = \frac{MP_K}{r}$

ii. At point e, iso-quant must convex towards the origin.

# Derivation of Equilibrium Conditions - Mathematical Approach

Maximizing output subject to the cost constraint can be presented as a formal constrained optimization problem as follows:

| | | | |
|---|---|---|---|
| Maximize | q = *f*(L,K) | ← | Objective function |
| Subject to | $C^0 = wL + rK$ | ← | Constraint |

Solution for this constrained maximization problem will determine the optimum factor combination which yield the maximum output.

We can solve this problem by using *Lagrangian Multiplier Method*, involve the following steps:

Step 1. Rewrite the constraint in the form

$$C^0 - wL - rK = 0$$

Step 2. Multiply the constraint by a constant λ which is the Lagrangian multiplier:

$$\lambda(C^0 - wL - rK) = 0$$

$\lambda$ can be defined as the 'marginal contribution of expenditure or marginal product of money in this example.

## Step 3. Form the 'composite' function:

$$z = q + \lambda(C^o - wL - rK)$$

The first condition for the maximization is that partial derivatives of the function with respect to L, K and $\lambda$ be equal to zero.

$$\frac{\partial z}{\partial L} = \frac{\partial q}{\partial L} - \lambda w \quad = 0 \qquad (1)$$

$$\frac{\partial z}{\partial K} = \frac{\partial q}{\partial K} - \lambda r \quad = 0 \qquad (2)$$

$$\frac{\partial z}{\partial \lambda} = C^0 - wL - rK = 0 \qquad (3)$$

Solving first two equations for λ

$$\lambda = \frac{\partial q / \partial L}{w} = \frac{MP_L}{w}$$

$$\lambda = \frac{\partial q / \partial K}{r} = \frac{MP_K}{r}$$

These two equations must be equal

$$\therefore \quad \frac{MP_L}{w} = \frac{MP_K}{r} \quad or \quad \frac{MP_L}{MP} = \frac{w}{r}$$

This is the first condition for output maximization.

For the second order condition for equilibrium require that marginal product curves have a negative slope.

Slope of the marginal product curves is the second derivative of the production function.

$$\frac{\partial^2 q}{\partial L^2} < 0 \quad \text{and} \quad \frac{\partial^2 q}{\partial K^2} < 0$$

togrther wit h

$$\left(\frac{\partial^2 q}{\partial L^2}\right)\left(\frac{\partial^2 q}{\partial K^2}\right) > \left(\frac{\partial^2 q}{\partial L \partial K}\right)^2$$

boarder Hessian determinant is an alternative way of taking the second order condition for equilibrium.

$$|H| = \begin{vmatrix} f_{11} & f_{12} & -w \\ f_{21} & f_{22} & -r \\ -w & -r & 0 \end{vmatrix} > 0$$

$f_{ij}$'s are the partial derivatives of the z.

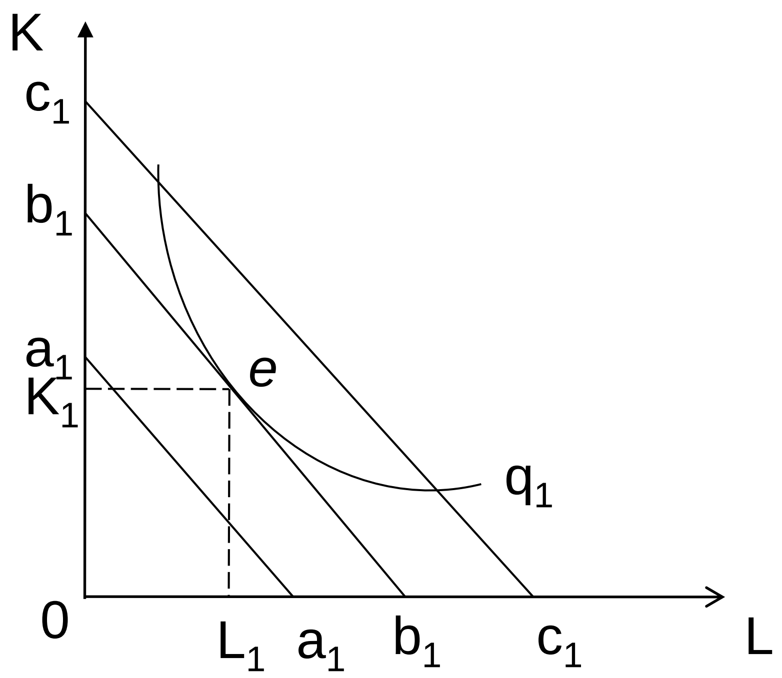
Given $Q=100L^{0.5}K^{0.5}$, C=1200, w = 30 and r = 40. Determine the optimum factor combination which maximize the output of the firm.

## Constrained cost minimization-Graphical approach

This is the dual problem of output maximization.

In this we find the conditions for minimum cost of production for a given level of output.

For the equilibrium, graphically, the iso-quant which shows the given level of output must be tangent to the lowest possible iso-cost line.



Equilibrium point is *e* which iso-quant that shows the desired output level tangent to the $b_1$-$b_1$ iso-cost line.

Points below *e* are desirable because they show lower cost but are not attainable for output $q_1$.

Points above *e* show higher cost. Hence, point *e* is the least-cost point.

At point *e*, slope of iso-quant = slope of iso-cost line

$$\frac{MP_L}{MP_K} = \frac{w}{r} = MRTS_{L,K}$$

By rearranging this, $$\frac{MP_L}{w} = \frac{MP_K}{r}$$

The equilibrium conditions are same as in the first case.

# Constrained Cost Minimization- Mathematical Approach

Objective function $C = wL + rK$

Constraint $q^0 = f(L, K)$

Producer's objective is to determine the least possible cost input combination that can be attained output level $q^0$.

To find the optimum factor combination, producer must minimize the cost of production subject to the output constraint.

Minimize $C = wL + rK$

Subject to $q^0 = f(L,K)$

Lagrangian (composite) function,

$$\Phi = (wL + rK) + \mu[q^0 - f(L,K)]$$

For the first order condition for cost minimization, partial derivative of Φ with respect to K, L and μ must equal to zero.

$$\frac{\partial \Phi}{\partial L} = w - \mu \frac{\partial f(L,K)}{\partial L} = 0 \qquad (1)$$

$$\frac{\partial \Phi}{\partial K} = r - \mu \frac{\partial f(L,K)}{\partial K} = 0 \qquad (2)$$

$$\frac{\partial \Phi}{\partial \mu} = q^0 - f(L, K) = 0 \qquad (3)$$

From (1) and (2)

$$w = \mu \frac{\partial q}{\partial L} \quad \text{and} \quad r = \mu \frac{\partial q}{\partial K}$$

Dividing through these expressions

$$\frac{w}{r} = \frac{\partial q / \partial L}{\partial q / \partial K} = MRTS_{L,K}$$ ← First order condition

Since $\frac{\partial q}{\partial L} = MP_L$ *and* $\frac{\partial q}{\partial K} = \mathrm{MP_K}$

At the equilibrium $$\frac{w}{r} = \frac{MP_L}{MP_K} = MRTS_{L,K}$$

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

Second order condition for cost minimization is similar to the output maximization

$$\frac{\partial^2 q}{\partial L^2} < 0, \quad \frac{\partial^2 q}{\partial L^2} < o \quad and \quad \left(\frac{\partial^2 q}{\partial L^2}\right)\left(\frac{\partial^2 q}{\partial K^2}\right) > \left(\frac{\partial^2 q}{\partial L \partial K}\right)$$

OR

$$|H| = \begin{vmatrix} f_{11} & f_{12} & -w \\ f_{21} & f_{22} & -r \\ -w & -r & 0 \end{vmatrix} > 0$$

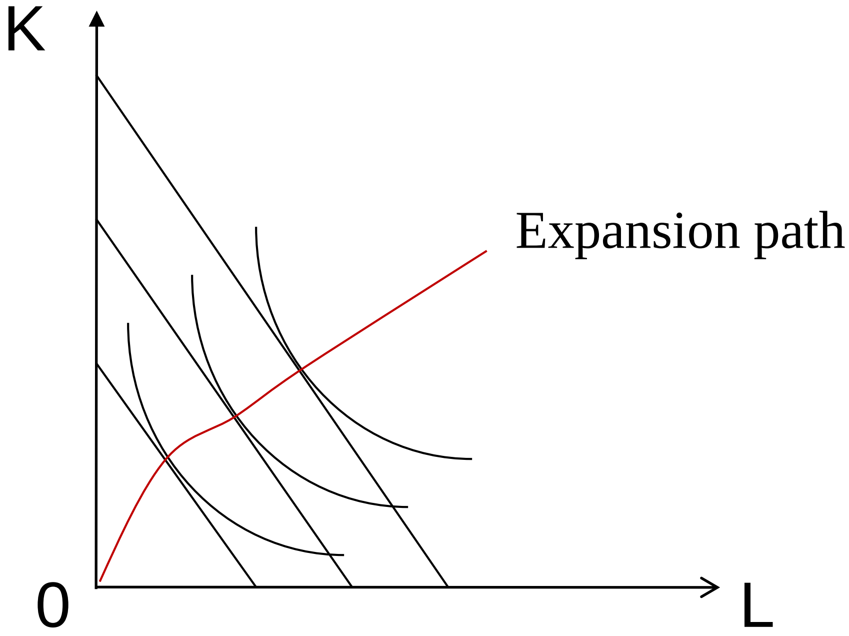
## Effect of Changes in Outlay on Equilibrium

The equilibrium change when factor prices and/or outlay change because these changes affect on the cost constraint.

Assume that total money (outlay) available increases keeping factor prices constant. In such a situation iso-cost line shift upward parallelly and the equilibrium is determined at the points it touches a higher iso-quant.

The shift of the iso-cost line continues further with increases in outlay. Thus producer’s equilibrium will be on higher and higher iso-quants.

The line or curve we gets by joining these equilibrium points passing through the origin called ‘expansion path’.




Expansion path can be interpreted as '*the locus of all equilibrium points when expenditure on inputs increases keeping input prices constant*'.

It shows the change of optimum factor combination when a firm expands its level of output at the given factor prices.

The expansion path is a very useful concept. It gives an idea of how input proportion changes with increases in expenditure of producer, input prices being constant.

Along the EP,

slope of iso-product curve = slope of iso-cost line

$$MRTS_{LK} = w/r$$

Expansion path for a linear homogeneous PFs will be a straight line showing constant proportion of the input used while increasing the level of output.

Expansion path for a non-homogeneous PFs will be a curve showing different factor proportions at different stages of production.

Derive the expansion path of the following function

$Q = AK^{0.5}L^{0.5}$

This is a linear homogenous PF. Why?

$$MP_L = \frac{\partial Q}{\partial L}$$

$$= 0.5AK^{0.5}L^{-0.5}$$

$$MP_K = \frac{\partial Q}{\partial K}$$

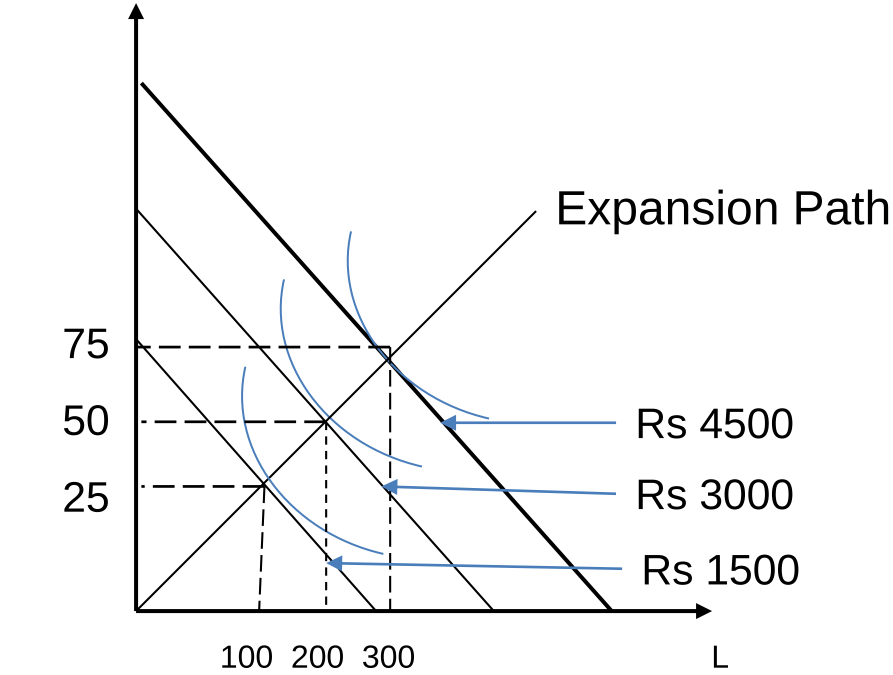
$$= 0.5AK^{-0.5}L^{0.5}$$

$$MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{0.5AK^{0.5}L^{-0.5}}{0.5AK^{-0.5}L^{0.5}} = \frac{K}{L}$$

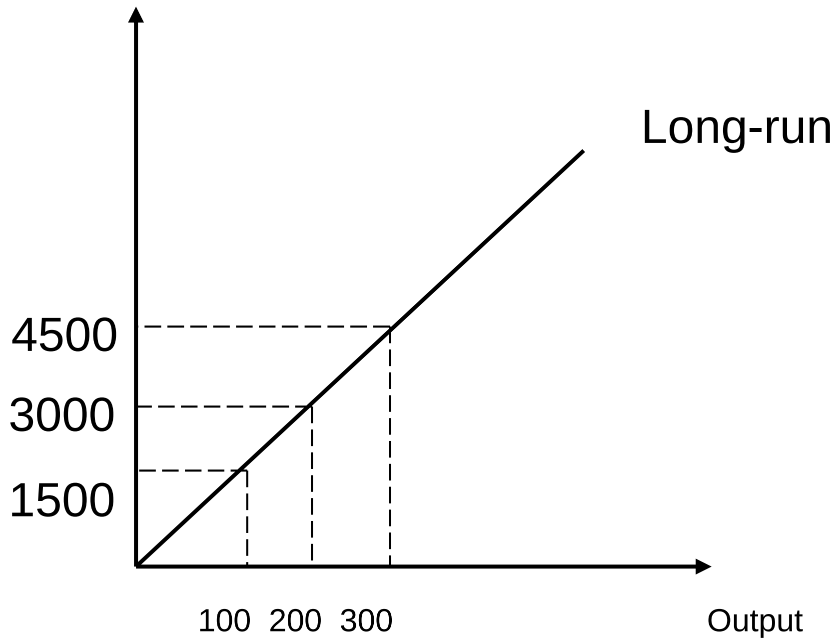
Since at optimal factor combination on EP, $MRTS_{LK}$=w/r

$$K/L = w/r$$

Given the factor prices, K/L ratio remain constant. Hence the EP is a straight line from the origin.




Expansion Path illustrates the lowest-cost combinations of L and K that can be used to produce each level of output in the long-run.




Points of TC measures the least cost of producing each level of output.

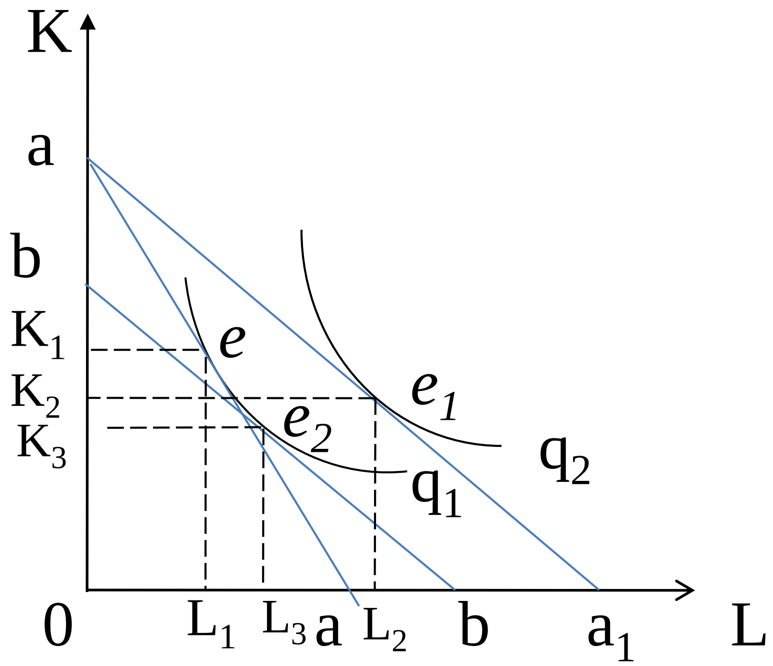
## Effects of the Change of Price of an Input Other Things Being Constant

When input prices change while other things remain constant, iso-cost line will shift and as a result the equilibrium position (input combination) will change.

Assume the price of L (w) decrease other things including price of K remains constant. Then, graphically iso-cost line will oscillate along the quantity axis of L (horizontal axis). With the reduced price, producer is capable of buying more of L than he bought earlier.

The producer will reach to a new equilibrium point in which oscillated iso-cost line tangent with higher iso-quant.

This oscillation of the iso-cost line continues further with decreases in price of L. Thus, producer's equilibrium will be on higher and higher iso-quants.




When price of a factor decrease the quantity use of that factor would increases. This increase of the quantity is considered as the result of two effects, namely *substitution effect* and *output effect*.

Equilibrium position has moved from *e* to $e_1$ on a higher iso-quant, which shows a higher output level ($q_2 > q_1$), as a result of the decreases of the price of L.

As a result of the decrease in price of L, optimum input combination has also changed from ($K_1L_1$) to ($K_2L_2$). Use of *L* has increased from $L_1$ to $L_2$. This is the *total effect* of the decrease in price of the input L.

Total Effect = Substitution Effect + Output Effect

*Substitution Effect*: When price of an input decreases while other things remain unchanged, producers tend to substitute it for the inputs that prices did not change. While output remaining constant, this change (increase) of use of input L is called substitution effect of decrease of the price of L.

This effect is always negative in the sense that a rise in the price of an input leads to reduce the use of that input and via-a-vis.

*Output Effect*: This shows the change of the level of input usage due to the change of output level, input prices remaining constant. This effect is normally positive.

According to the above graph,

$L_1$-$L_2$ = total effect

$L_1$-$L_3$ = substitution effect

$L_3$-$L_2$ output effect

# Technical Progress

Technology is assumed to be constant.

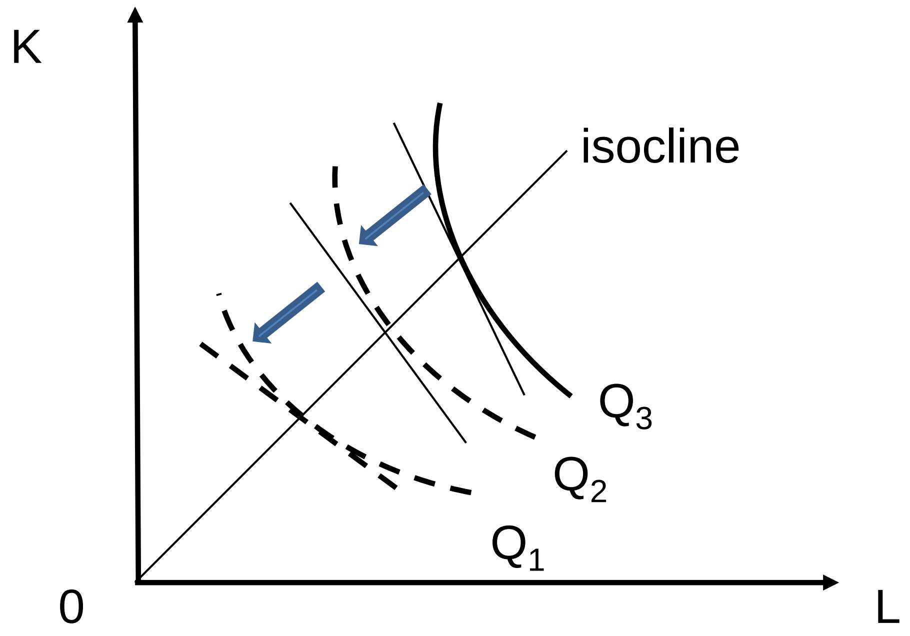
In practice it is changed due to various factors including innovation, education etc.

Considering the effect of technological progress on the rate of the substitution of the factors of Production or on the capital-labor ratio, J. R. Hicks defined three type of technological progress.

# Capital Deepening technical progress

If the technical progress happens to strengthen the efficiency of capital as such marginal product of capital by more than the labor, it is called capital deepening technical progress.

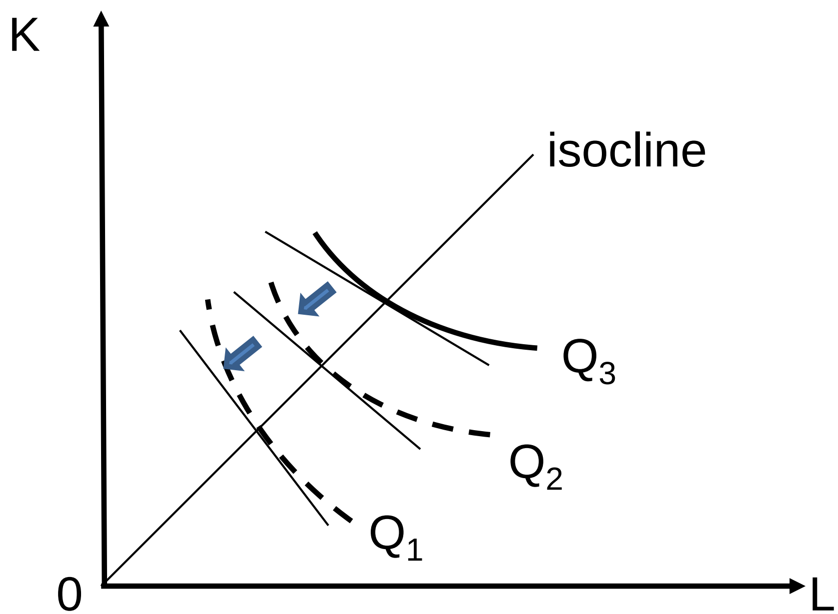
In such progress, along a line through the origin which the K/L ratio constant, $MRTS_{L,K}$ decreases: slope of the shifting iso-quants become less steep.



# Labor deepening technical progress

If the technical progress happens to strengthen the efficiency of labor as such marginal product of labor than of capital, it is called labor-deepening technical progress.

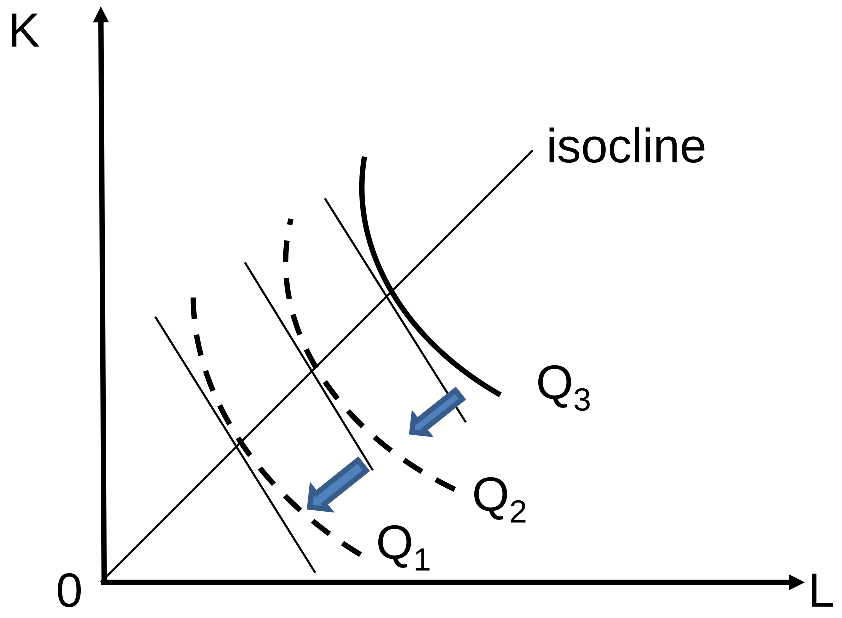
In such progress along a line through the origin which the K/L ratio constant, $MRTS_{L,K}$ increases: slope of the shifting iso-quant is steep.



# Neutral technical progress

If the technical progress happens to strengthen the efficiency of both capital and labor inputs equally, and marginal product of both factors increase by same percentage, it is called neutral technical progress.

In such progress, along a line through the origin which the K/L ratio constant, $MRTS_{L,K}$ remains constant: The iso-quant shifts parallel to itself.



# Multi-Product Firm: Choice of Product Mix

So far we discussed the behavior of a firm in deciding optimum factor combination for producing a single product.

In the real world many are multi-product firms!

Thus, the multi-product firms have to decide the optimum product combination.

This is important because the firms possess only a limited amount of resources for production process.

Table below shows the alternative production possibilities of a firm produce two product X and Y, assuming

i. the firm has a given amount of resources
ii. is operating under a given technology

| Production possibilities | Product X (‘00) | Product Y (’00) |
|---|---|---|
| A | 0 | 15 |
| B | 1 | 4 |
| C | 2 | 12 |
| D | 3 | 9 |
| E | 4 | 5 |
| F | 5 | 0 |

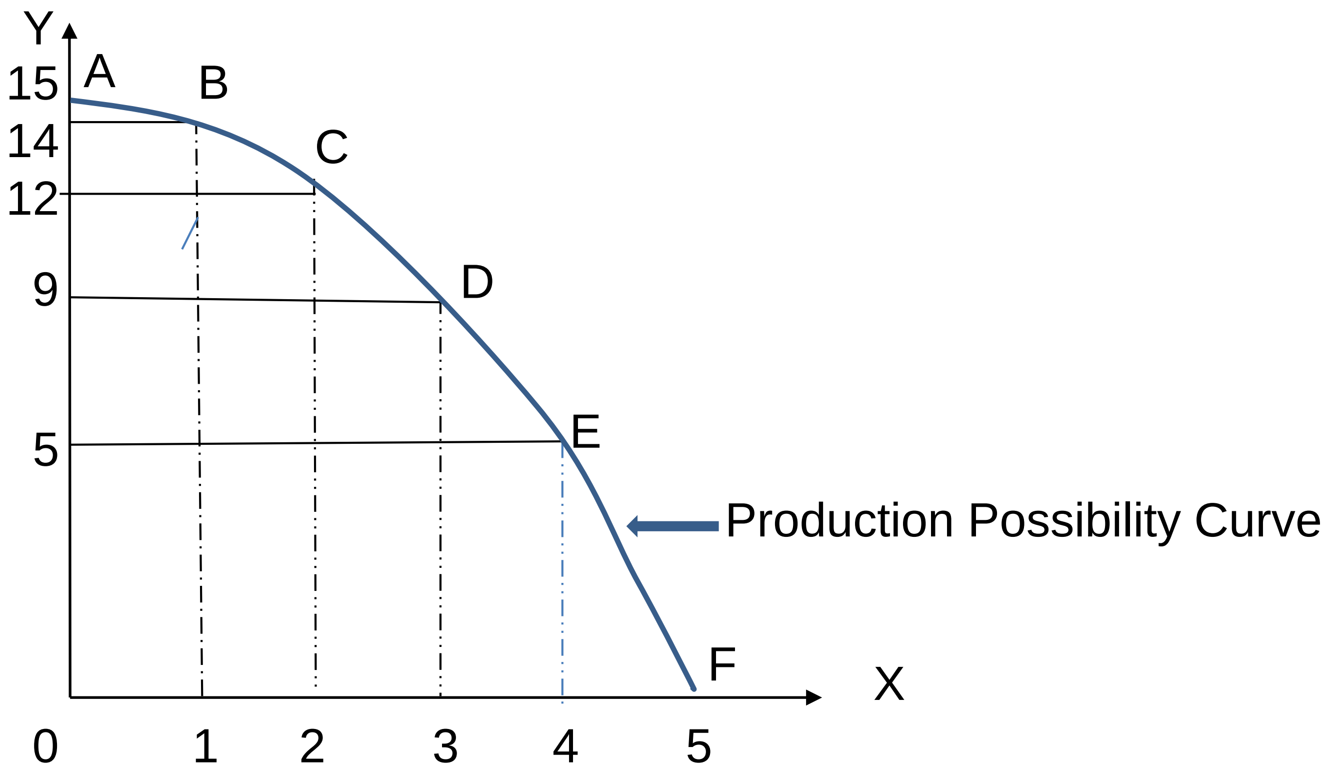
If firm employed all resources on Y he can produced 15 hundred units.

If he employed all resources on X he can produce 5 hundred units.

Within this extremes, he can produce given product mix.

The figures in the table revel that to produce one extra unit of X he has to scarifies increasing amount of Y.

These alternative production possibilities can be depicted in a graph:



The curve A-F shows the various combinations of the two products that the firm can produce with the given amount f resources and under the given technology.

It is called Production Possibility Curve or Product Transformation Curve.

Alternative product combinations of two products are represented along the PPC.

PPC is concave to the origin.
This implies that to increase the amount of one product with the given volume of resources, the producer has to scarifies some amount from the other product.

When he move from A to F on PPC, it sacrifices some amount of Y for having more of X.
When he move from F to A on PPC, it sacrifices some amount of X for having more of Y.

In moving along the PPC, it transform one product into the other.

The rate at which one product transform into another, resources keeping unchanged, is called Marginal Rate of Transformation ($MRT_{XY}$).

When moving along the PPC, $MRT_{XY}$ increases. This is the reason to PPC concave to the origin.

MRTS at any point on PPC is given by the slope of the curve at that point.

When the firm fully utilized the resources, the optimum product combination must lie some where on the PPC but not inside.

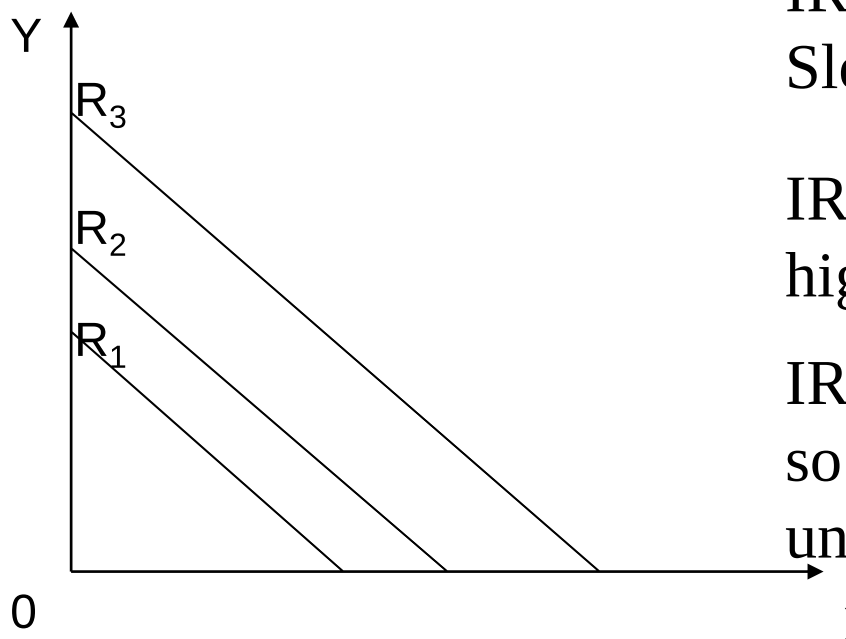
## Iso-Revenue Lines

IRL is an important tool to determine the optimum products combination.

IRL shows the different product combinations which earn the same revenue.

Given the fixed prices of X and Y, the IRL can be written as

$$R = P_x Q_x + P_y Q_y$$




IRL is a straight line.
Slope = $P_y/P_x$

IRL away from the origin shows higher revenue.

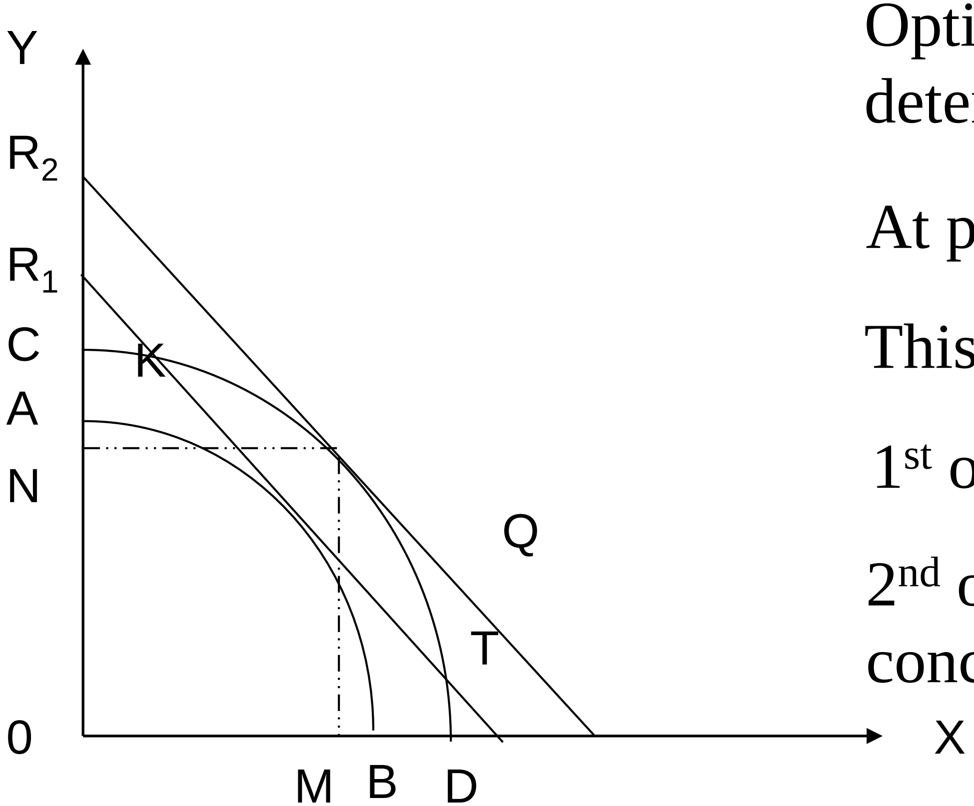
IRLs are parallel to each other so as to price ratio remain unchanged.

# Optimum Product Combination

It assumes that the aim of the producer is to maximized the profit.

The profit will be maximized when the firm maximize its revenue.

Optimum product mix can be obtained graphically using PPC and IRL.




Optimum product combination determine at point Q,

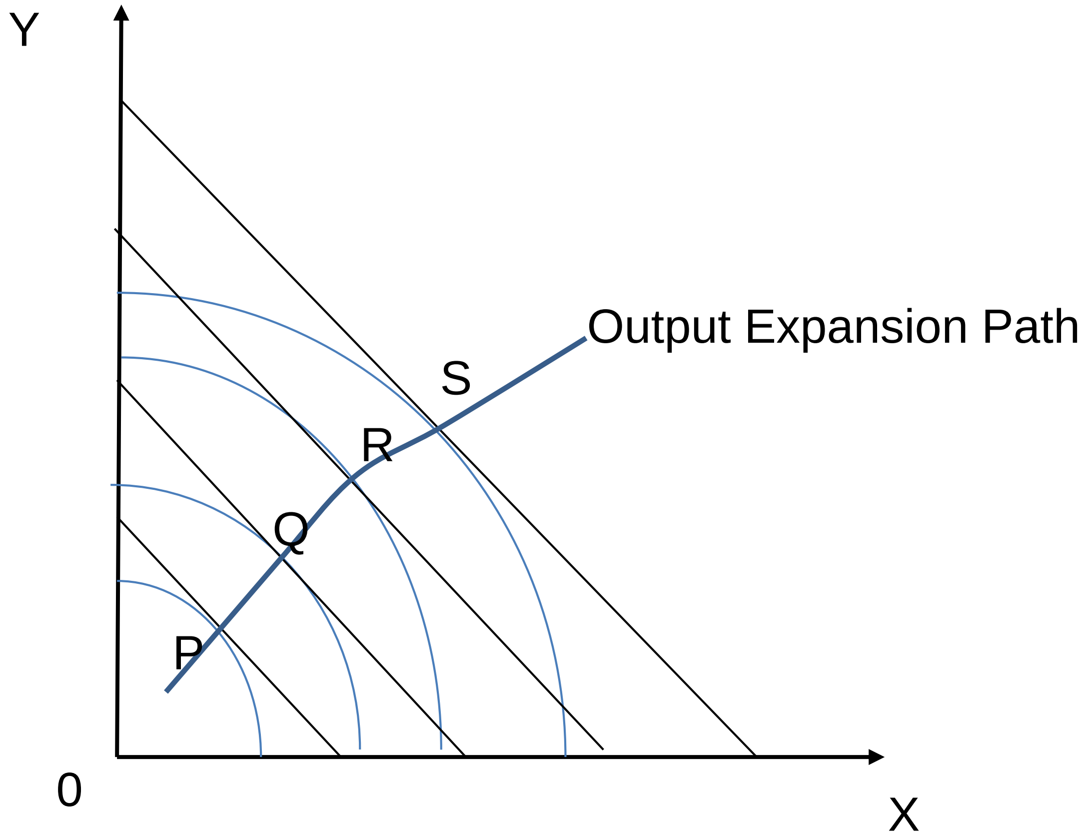
At point Q, $MRT_{XY}=P_y/P_X$

This is not fulfilled as points K and T

$1^{st}$ order condition $MRT_{XY}=P_y/P_X$

$2^{nd}$ order condition PPC must be concave to the origin.

# Output Expansion Path




Locus of all the revenue maximizing product combinations with the varying amount of resources.

The firm expands its output along this path as its resources increases.