## The Lesson 7

## Network Analysis for project Management

## Introduction

The purpose of lesson six is to study how network techniques could use for project planning, monitoring and management. The session will explain basic characteristics of network analysis and main network analytical tools such as Project Evaluation and Review Techniques (PERT) and Critical Path Analysis (CPA) methods.

Due to some limitations of using Gantt charts in project scheduling i.e. not indicate inter relationships between project tasks/activities and impacts of delaying tasks or of shifting resources, network analysis is used as planning technique for project management.

### 7.1 Objectives of Network Analysis

Network analysis is used to serve several objectives (Sharma, A. K. 2006).

1. Minimization of total time
2. Minimization of total costs
3. Minimization of cost for a given total time
4. Minimization of time for a given cost
5. Minimization of idle resources

### 7.2 Diagrams and Networks

Three main diagramming methods are used in project planning and management (Nicholas, J. M, 2001).

1. Precedence Diagramming Method (PDM)
2. Program Evaluation and Review Techniques (PERT)
3. Critical Path Method (CPM)

### 7.2.1 Precedence Diagramming Method

PDM shows the logical relationship of major elements of work package (tasks) to be performed as "precedence" or to be completed before others as predecessor. It is also important to distinguish what activities are the successors and do at the same time. For example if we assess the activities of a person who wake up in the morning and getting ready for work, sequence of his activities could list as prior activities to be performed to do next activities. Thus considering immediate Precedence of work and time spent for each activity, whole process can present orderly as shown in table 7.1.

Table 7.1: Activities and Immediate Precedence

| Activity | Immediate Precedence | Duration (seconds) |
| :--- | :--- | :--- |
| A. Get undressed | - | 60 |
| B. Take shower | A | 600 |
| C. Put on underwear | B | 40 |
| D. Dry, brush hair | B | 350 |
| E. Put on shirt | C | 150 |
| F. Put on pants | C | 60 |
| G. Put on shocks | C | 45 |
| H. Put on tie | E | 180 |
| I. Put on shoves | F, G | 100 |
| J. Put on jacket | H, I, D | 15 |

Quoted from Nicholas, J. M. (2001)

Logic diagram on the above activities and immediate precedence is presented in diagram 7.1.

Dig 7.1: Logic Diagram


## Activity on Nodes Diagrams

Network diagrams are used to present the sequence of activities and events associated with the project. According to nodes diagrams method, nodes (circles) are used to represent activities and the events represent as an instant in time signifies as start and finish of activity. (Dig. 7.2). Thus activity is a work task to be implemented. It can be a part of WBS, cluster of work package or individual job that execute with resources and time. Event is an instant in time indicate start and finish.

Dig. 7.2: Activity on Node


## Activity on Arrow Diagrams (AOA method)

According to AOA method, the activity is represented as directed line segments that called on arrow or arc between two nodes (circles). As shown in diagram 7.3, nodes represent the events of start and finish and arrow line indicates the activity
and the time. Note that activity is presented above the line and time is presented below the line.

Dig. 7.3: Activity on Arrow Diagrams


An Example
Both AON and AOA formats use as activity oriented networks to show (describe) projects in terms of tasks or jobs in planning and implementation.

If you were given information about the project as shown in table7.2, you can prepare a network either using AON method or AOA method.

Table 7.2 Activity, Immediate predecessor and Time

| Activity | Immediate predecessor | Time (Days) |
| :--- | :--- | :--- |
| A | - | 06 |
| B | A | 09 |
| C | A | 08 |
| D | B, C | 04 |
| E | B, C | 06 |

Dig. 7.4: AON Method


Note that AON networks construct without use of dummy and it is easier and simple to construct. As shown in Dig. 7.4, nodes used to indicate activity and time, and arrows used to indicate relationship between events and activities.

When use AOA method, node represents events and arrows shows the activity. Each events were numbered including start and finish (Dig 7.5).

Dig. 7.5: AOA Method


AOA method is used often by project managers because it suit for PERT and CPM procedures to indicate the relationship of project activities, duration and costs.

### 7.2 2 Program Evaluation and Review Techniques (PERT)

PERT was developed by U.S. Navy's Polaris Missile System Program in 1958. Though initially it was developed for defense projects, now it widely uses as planning and controlling projects. It emphasis the uncertainty of project finish by considering variation of project time in terms of three scenarios as follows.

Optimistic Time: Minimum Time required for an activity to complete, the situation where everything goes well. (See point a in figure 7.1)

Most likely (Normal) Time: It is the normal time to complete the task. It assumes that no extra time or personnel required. (See point m in figure 7.1)

Pessimistic Time: The maximum time that activity could take. It is the worst time that could be expected if everything went wrong. (See point $b$ in figure 7.1)

Fig. 7.1: Probability Distribution of Time
Activity


As shown above Probability Distribution of project time is based on the three possible scenarios as optimistic, ml and pessimistic time distribution. Thus expected time (te) and of each activity could be calculated using following formulas.

$$
\mathrm{te}=\mathrm{ta}+4 \mathrm{t}_{\mathrm{m}}+\mathrm{tb}
$$

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$$
\begin{aligned}
& \mathrm{te}=\text { expected time } \\
& \mathrm{ta}=\text { Optimistic time } \\
& \mathrm{tm}=\text { Most likely time }(\mathrm{ML}) \\
& \mathrm{tb}=\text { Pessimistic time }
\end{aligned}
$$

According to time distribution of fig 6.1, the average or expected time (te)is

$$
=3+4(5)+13 / 6=6 \text { days }
$$

The Variance also counted as follows;

$$
\mathrm{V}=(b-a) 2
$$

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Thus Variance of above distribution is

$$
\mathrm{V}=\frac{(13-3)}{6}=(1.67 \text { days }) 2=2.78 \text { days }
$$

The following Example indicates details of activities of a construction project. If the management want to calculate expected time under three possible time scenarios i.e. optimistic, most likely and pessimistic time, how do you calculate the expected time for the project?

Table 7.4 Calculate Expected Time

| Activity | Immediate <br> Predecessor | Optimistic <br> Time (a) | Most likely <br> Time (m) | Pessimistic <br> Time = (b) | Expected Time <br> te $=\frac{a+4 m+b}{6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| A | - | 03 | 04 | 05 | 04 |
| B | A | 01 | 02 | 03 | 02 |
| C | A | 02 | 03 | 04 | 03 |
| D | C | 01 | 02 | 03 | 02 |
| E | A | 03 | 05 | 07 | 05 |
| F | C | 04 | 05 | 12 | 05 |
| G | B,D | 05 | 04 | 11 | 04 |
| H | E | 92 | 04 | 06 | 04 |
| I | F,G | 02 | 03 | 04 | 03 |
| J | H.I | 04 | 06 | 08 | 06 |
|  |  |  |  |  |  |

[^0]The network analysis based on the table 7.4 is presented by a diagram below.
Dig. 7.6: Network Analysis


### 7.3 Critical Path Method (CPM)

CPM method was introduced in 1957 by DuPont Company and expanded to include Remington Rand and Mouchy Associates. The CPM emphasis to minimize the cost while reducing time period. Thus it concern more on the resource availability to accompany the project task by managing time and cost (See fig. 7.2).

Fig. 7.2: The relationship of Time and Cost

$\mathrm{Cn}=$ Normal Cost
$\mathrm{Cc}=$ crash cost
$\mathrm{Tn}=$ Normal Time
Tc $=$ crash time
According to above equation and the statistics in the figure 7.2, additional cost requirement ( $\mathrm{Cc}-\mathrm{Cn}$ ) to reduce duration from $\mathrm{Tn}-\mathrm{Tc}$ ( $8-5$ weeks) is calculated as follows.

Cc-Cn / Tn-Tc
$=18-9 / 8-5=9 / 3$
$=3$

## Critical path

Critical path is the longest path in a network. It is the order of activities or task that indicate maximum time to complete the project. It is important because it reflects earliest start time and earliest finish time. So it helps to identify faults of the path that would affect to delay the project. Therefore, both PERT and CPM methods count critical path in the network analysis. Four type of durations are considered in critical path analysis. It includes;

## 1. Earliest Start Time (EST);

It is the time, which indicates earliest start time of activity without delaying the total project time
2. Earliest Finish Time (EFT):

It is the time, which indicates earliest finish time of activity without delaying the total project time
3. Latest Start Time (LST):

The time, which indicates latest start time of activity without delaying the total project time

## 4. Latest Finish Time (LFT):

It is the time, which indicates latest finish time of activity without delaying the total project time

If we consider expected time calculated in table 7.4, EST and EFT for each task could be calculated as follows;

For activity A, no early start work and therefore EFT is 04 days. For activity B, EST is 4 because it start after activity and it require 2 days to finish. Thus EFT is $4+2=6$. The activity C also start after activity A and hence EST is 4 and since activity C require 3 days, its EFT is $4+3=7$. Thus EFT for all activities can be calculated by counting EST and time require for each activity. According to table 7.4 and diagram 7.6, EST for last activity J is 17 days and its EFT is 23 days. It indicates the longest path of the projected to be completed. EST and EFT is mentioned above the arrow. Since the last activity and the last event are considered as total time required to complete project, LST and LFT of each and every activities counted by reducing required time for each activity from EFT. For instance, EFT of the last activity is 23 and it consider as the LFT. Thus LST for activity J is counted by reducing required time for activity J as follows (23-6 = 17). Similarly, LST and LFT for each activity could be calculated. LST and LFT is mentioned below the arrow (Table 7.5).

Counting early and late perspectives indicates clearly critical path. Thus critical and non-critical activities identify easily by counting slack, which reflects time period for which an activity can be delayed without causing troubles in completion of project. It may be positive or negative. When slack become negative it recognize as critical activity and when it becomes positive it recognize as non-critical activity If we consider the same example in accordance with slack, the diagram analysis could be illustrated as follows (See diagram 7.7). The critical path is denoted by double line or darker to distinguish it from non-critical activities.

Table 7.5: Counting Slack according to Network

| Activity | EST | EFT | LST | LFT | Slack* |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A (1-2) | 0 | 04 | 0 | 04 | 0 |
| B (2-4) | 4 | 06 | 07 | 09 | 03 |
| C (2-3) | 4 | 07 | 04 | 07 | 0 |
| D (3-4) | 7 | 09 | 07 | 09 | 0 |
| E (2-5) | 4 | 09 | 08 | 13 | 04 |
| F (3-6) | 7 | 13 | 08 | 14 | 1 |
| G (4-6) | 9 | 14 | 09 | 14 | 0 |
| H (5-7) | 9 | 13 | 13 | 17 | 04 |
| I (6-7) | 14 | 17 | 14 | 17 | 0 |
| J (7-8) | 17 | 23 | 17 | 23 | 0 |
|  |  |  |  |  |  |

*Counting slack $\longrightarrow$ LST- EST, LFT- EFT
Dig. 7.7: Critical Path Analysis




[^0]:    Quoted from Pathirage, J. M. P. (2013)

